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# The Bohm-Vigier Subquantum Fluctuations and Nonlinear Field Theory

YU. P. RYBAKOV

*Institut Henri Poincaré, 11, rue Pierre et Marie Curie, Paris 5è, France*†

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*Abstract*

Utilising as starting point the double solution theory of Louis de Broglie and the Bohm–Vigier hypothesis of subquantum fluctuations, an attempt is made to give an example of the nonlinear field theory in which the 'guidance' theorem of de Broglie can be realised. The simplest scalar model is considered.

## 1. *Introduction*

In the de Broglie double solution theory (de Broglie, 1956) an elementary particle is described by a regular solution ($u$-wave) of the underlying nonlinear field equations. The field $u(\mathbf{r}, t)$ is supposed to be square summable. Hence, $u$ vanishes at infinity

$$\lim_{r\to\infty} u(\mathbf{r}, t) = 0$$

For this reason the field equations may be linearised in the region of space outside the particle, where $u(\mathbf{r}, t)$ is sufficiently small. In this region there exists a new class of solutions ($v$-waves). It is assumed that only $v$-waves must be responsible for the wave properties of elementary particles and coincide with the corresponding solutions of the wave mechanical equations, the normalisation factor being omitted.

For the explanation of the fact that $v$-waves may play the role of probability amplitudes Bohm & Vigier (1954) introduced the subquantum medium ('thermostat caché') which had to be the source of subquantum fluctuations. In the proof of their theorem they used the hydrodynamical analogy and, in particular, the continuity equation $\partial_\mu j^\mu = 0$ with positive density $j^0 > 0$.

The hypothesis of the subquantum medium plays a fundamental role in the de Broglie thermodynamics of the isolated particle (de Broglie, 1964, 1968).

The following is an attempt to find new applications of the Bohm–Vigier hypothesis to the nonlinear field theory. Namely, it is supposed that the

† Present address: Patrice Lumumba University, the Chair of Theoretical Physics, Moscow, B-302, U.S.S.R.

existence of subquantum fluctuations must change the differential characteristics of the underlying field equations.

In fact, let the field $u$ satisfy a nonlinear differential equation. Since the characteristics of any differential equation give the propagation law of small perturbation front, it is sufficient to consider the linearised field equation.

Following de Broglie (1967), one can represent the front velocity $\mathbf{V}$ as a sum of its regular and random parts:

$$\mathbf{V} = \mathbf{v}_{\text{reg}} + \mathbf{v}_{\text{rand}} \tag{1.1}$$

Since by definition $\mathbf{v}_{\text{reg}}$ is an average front velocity, we have

$$\langle \mathbf{V} \rangle = \mathbf{v}_{\text{reg}}$$

and therefore

$$\langle \mathbf{V}^2 \rangle = \mathbf{v}_{\text{reg}}^2 + \langle \mathbf{v}_{\text{rand}}^2 \rangle \tag{1.2}$$

Now it is natural to assume that

$$\langle \mathbf{V}^2 \rangle = c^2$$

where $c$ is the velocity of light. If now we introduce the 'regular' and 'random' coordinates by putting

$$\mathbf{v}_{\text{reg}}^2 \equiv \left(\frac{d\mathbf{r}}{dt}\right)^2, \qquad \langle \mathbf{v}_{\text{rand}}^2 \rangle \equiv \left(\frac{d\boldsymbol{\xi}}{dt}\right)^2$$

we shall obtain the following characteristics for our field equations:

$$c^2\, dt^2 = d\mathbf{r}^2 + d\boldsymbol{\xi}^2 \tag{1.3}$$

Thus, the field function $u$ must depend not only on the ordinary space-time coordinates $x^\mu$, $\mu = 0, 1, 2, 3$, but also on the complementary 'random' ones $\xi^j$, $j = 1, 2, 3$, which are just to characterise the subquantum fluctuations. The possible corollaries of this hypothesis will be considered in the simplest case of a neutral scalar field.

## 2. *Scalar Field Model. Uniqueness of the Vacuum*

Let $\varphi(x, \xi)$ be a neutral scalar field function. In order to distinguish differentiation by $x^\mu$ and $\xi^j$, it will be convenient to write:

$$\frac{\partial}{\partial \xi^j} \equiv \boldsymbol{\partial}_j \equiv \nabla_j; \qquad \boldsymbol{\partial}_j^2 \equiv \Delta \tag{2.1}$$

Next, it is obvious that all the physical quantities must be obtained as averages over $\xi$-space. This means that, for example, the hamiltonian may contain the integrals of the kind

$$\int d^3\,\xi \rho(\xi)\,\varphi^2$$

Since we consider only regular field functions $\varphi(x,\xi)$, we may choose $\rho(\xi) = \xi^2$ and introduce in the Hamiltonian the integral

$$\mathscr{T} \equiv \int d^3\xi(\xi^2\varphi^2) \tag{2.2}$$

Thus, we may consider the following field action:

$$S = -\tfrac{1}{2}\int d^4x\mathscr{F}(\mathscr{T}) - \tfrac{1}{2}\int d^4x\, d^3\xi[\partial_\mu\varphi\,\partial^\mu\varphi + (\nabla\varphi)^2 - \lambda_0\varphi^2] \tag{2.3}$$

where $\mathscr{F}(\mathscr{T})$ is an unknown nonlinear function.

Then the variational principle gives the following field equation:

$$(\Box + \triangle + \lambda_0 - \xi^2\mathscr{F}'(\mathscr{T}))\varphi = 0 \tag{2.4}$$

First of all we must find the solution of (2.4) which corresponds to a no-particle state, i.e. vacuum. In this case the field function need not depend on $x^\mu$, that is $\varphi = \varphi_0(\xi)$.

It turns out that one can choose the constant $\lambda_0$ so that the vacuum solution will be unique and spherically symmetrical. In fact, the equation for $\varphi_0$ is

$$\varphi_0'' + \frac{2}{\xi}\varphi_0' + \lambda_0\varphi_0 - \xi^2\varphi_0\mathscr{F}'(\mathscr{T}_0) = 0 \tag{2.5}$$

where

$$\mathscr{T}_0 \equiv \mathscr{T}[\varphi_0]$$

It will be shown later that the number $\mathscr{T}_0$ can be determined uniquely, provided that the suitable function $\mathscr{F}(\mathscr{T})$ is chosen.

The equation (2.5) has the evident simplest solution of the form:

$$\varphi_0 = C\exp\{-\tfrac{1}{6}\lambda_0\xi^2\} \tag{2.6}$$

if

$$\mathscr{F}'(\mathscr{T}_0) = \tfrac{1}{9}\lambda_0{}^2 \tag{2.7}$$

The relation (2.7) gives a single value of $\lambda_0 > 0$, which provides the uniqueness of the vacuum solution. By direct computation of (2.2) one can also find the value of the constant $C$ in (2.6):

$$C^2 = 2\mathscr{T}_0\lambda_0^{5/2}/27\sqrt{3}\,\pi^{3/2}$$

Now we must determine $\mathscr{T}_0$. Note for this that the vacuum state must be relativistically invariant and therefore its energy must vanish, i.e.

$$E_0 = 0 \tag{2.8}$$

It turns out that this condition is sufficient to determine $\mathscr{T}_0$. In fact, multiplying (2.5) by $\varphi_0$ and integrating over $\xi$-space, one finds the following identity:

$$\mathscr{T}_0\mathscr{F}'(\mathscr{T}_0) + \int d^3\xi[(\nabla\varphi_0)^2 - \lambda_0\varphi_0{}^2] = 0 \tag{2.9}$$

with the aid of which the vacuum energy expression can be transformed as follows:

$$E_0 = \tfrac{1}{2}\int d^3x\mathscr{F}(\mathscr{T}_0) + \tfrac{1}{2}\int d^3x\, d^3\xi[(\mathbb{\nabla}\varphi_0)^2 - \lambda_0\varphi_0{}^2]$$
$$= \tfrac{1}{2}\int d^3x[\mathscr{F}(\mathscr{T}_0) - \mathscr{T}_0\mathscr{F}'(\mathscr{T}_0)]$$

It is seen that (2.8) holds if $\mathscr{T}_0$ satisfies the equation

$$\mathscr{F}(\mathscr{T}_0) = \mathscr{T}_0\mathscr{F}'(\mathscr{T}_0) \tag{2.10}$$

It should be noticed that the proposed scheme will be consistent if and only if the solution of (2.10) is unique and compatible with (2.7). Then the problem of vacuum state will be solved.

### 3. *u- and v-waves. Mass Spectrum*

Consider an arbitrary regular solution of (2.4), which corresponds to a $u$-wave and satisfies the boundary conditions:

$$\lim_{r\to\infty}\varphi(x,\xi) = \varphi_0(\xi); \qquad \lim_{\xi\to\infty}\varphi(x,\xi) = 0 \tag{3.1}$$

Prove that if such a solution exists, then its field energy is strictly positive. The field energy may be written as follows:

$$E = \tfrac{1}{2}\int d^3x\mathscr{F}(\mathscr{T}) + \tfrac{1}{2}\int d^3x\, d^3\xi[(\nabla\varphi)^2 + (\mathbb{\nabla}\varphi)^2 + (\partial_0\varphi)^2 - \lambda_0\varphi^2] \tag{3.2}$$

Since $E$ is the integral of motion, one can put

$$E = \langle E\rangle_t$$

where by definition

$$\langle f\rangle_t \equiv \lim_{T\to\infty}\frac{1}{T}\int_0^T f(t)\,dt \tag{3.3}$$

Multiplying (2.4) consecutively by $\varphi$, $(\mathbf{r}\cdot\nabla)\varphi$ and $(\boldsymbol{\xi}\cdot\mathbb{\nabla})\varphi$ and then carrying out the $\mathbf{r}$, $\boldsymbol{\xi}$-integrations, complemented by the operation (3.3), one can obtain the following three identities:

$$\int d^3x\langle\mathscr{T}\mathscr{F}'(\mathscr{T}) + \int d^3\xi[(\nabla\varphi)^2 + (\mathbb{\nabla}\varphi)^2 - (\partial_0\varphi)^2 - \lambda_0\varphi^2]\rangle_t = 0$$
$$\int d^3x\langle\mathscr{F}(\mathscr{T}) + \int d^3\xi[\tfrac{1}{3}(\nabla\varphi)^2 + (\mathbb{\nabla}\varphi)^2 - (\partial_0\varphi)^2 - \lambda_0\varphi^2]\rangle_t = 0$$
$$\int d^3x\langle\tfrac{5}{3}\mathscr{T}\mathscr{F}'(\mathscr{T}) + \int d^3\xi[(\nabla\varphi)^2 + \tfrac{1}{3}(\mathbb{\nabla}\varphi)^2 - (\partial_0\varphi)^2 - \lambda_0\varphi^2]\rangle_t = 0$$

the boundary conditions (3.1) and the equations (2.9) and (2.10) being taken into account.

These identities may be rewritten in the more simple form:

$$\int d^3 x \langle \mathscr{F}(\mathscr{T}) - \int d^3 \xi [\tfrac{2}{3}(\nabla \varphi)^2 + (\mathbb{V}\varphi)^2] \rangle_t = 0 \tag{3.4a}$$

$$\int d^3 x \langle \mathscr{T} \mathscr{F}'(\mathscr{T}) - \int d^3 \xi (\mathbb{V}\varphi)^2 \rangle_t = 0 \tag{3.4b}$$

$$\int d^3 x\, d^3 \xi \langle (\nabla \varphi)^2 + 2(\mathbb{V}\varphi)^2 - (\partial_0 \varphi)^2 - \lambda_0 \varphi^2 \rangle_t = 0 \tag{3.4c}$$

Using (3.4a, b, c), the following expression for the field energy can be easily obtained:

$$E = \int d^3 x\, d^3 \xi \langle (\partial_0 \varphi)^2 + \tfrac{1}{3}(\nabla \varphi)^2 \rangle_t \tag{3.5}$$

which proves our statement.

Now we may treat the asymptotic behaviour of the field function ($v$-waves). With this intention put

$$\varphi = \varphi_0 + \psi$$

where by virtue of the boundary conditions (3.1) $\psi \ll \varphi_0$. Then $\psi$ satisfies the following linear equation:

$$(\Box + \mathbb{A} + \lambda_0 - \xi^2 \mathscr{F}'(\mathscr{T}_0))\psi = 2\xi^2 \varphi_0 \mathscr{F}''(\mathscr{T}_0) \int d^3 \xi (\varphi_0 \psi \xi^2) \tag{3.6}$$

Since the coefficients in (3.6) depend only on $\xi$, one may separate the variables by putting

$$\psi(x, \xi) = v(x)\, w(\xi) \tag{3.7}$$

The substitution of (3.7) into (3.6) gives:

$$(\Box - m^2) v = 0 \tag{3.8}$$

$$(\mathbb{A} + \lambda_0 + m^2 - \xi^2 \mathscr{F}'(\mathscr{T}_0))\, w = 2\xi^2 \varphi_0 \mathscr{F}''(\mathscr{T}_0) \int d^3 \xi (\varphi_0 w \xi^2) \tag{3.9}$$

where $m^2$ is the separation constant.

It is seen that $v(x)$ satisfies the well-known Klein–Gordon equation for a scalar particle of mass $m$ ($\hbar = c = 1$). The correspondent mass spectrum is determined by the equation (3.9). It may be shown that the mass spectrum will be real ($m^2 \geqslant 0$) if

$$\mathscr{F}''(\mathscr{T}_0) \geqslant 0 \tag{3.10}$$

In fact, from the equation (3.9) we obtain

$$m^2 \int d^3 \xi w^2 = 2\mathscr{F}''(\mathscr{T}_0) \left[\int d^3 \xi (\varphi_0 w \xi^2)\right]^2 + \int d^3 \xi [(\mathbb{V} w)^2 - \lambda_0 w^2 + \xi^2 \mathscr{F}'(\mathscr{T}_0) w^2] \tag{3.11}$$

At the same time, as it is seen from (2.5) and (2.6), $\lambda_0$ is the lowest eigenvalue of the operator

$$\hat{L} \equiv -\mathbb{A} + \xi^2 \mathscr{F}'(\mathscr{T}_0)$$

Therefore

$$\int d^3\xi[(\nabla w)^2 - \lambda_0 w^2 + \xi^2 \mathscr{F}'(\mathscr{T}_0) w^2] = \int d^3\xi[w(\hat{L} - \lambda_0) w] \geqslant 0 \quad (3.12)$$

Hence, as follows from (3.11) and (3.12), $m^2 \geqslant 0$ if $\mathscr{F}''(\mathscr{T}_0) \geqslant 0$.

## 4. *The 'Guidance' Theorem. Discussions*

Now we can pass to the well-known 'guidance' theorem of Louis de Broglie. This theorem states that the system of $u$- and $v$-waves must be self-consistent, that is the behaviour of $u$-wave must be determined by the form of $v$-wave and conversely (de Broglie, 1956).

The connection of $u$, $v$-waves will appear if we make the following natural assumption. Namely, suppose that $v$-waves are generated by $u$-waves. It means that we must impose the following initial condition

$$\lim_{t \to -\infty} v(x) = 0$$

Then the part of general solution of (3.6), which contains $v$-waves, can be written with the aid of the retarded Green function as follows:

$$\psi(x) = \sum_s w_s(\xi) \int d^4k \exp(ikx) g_s(k) \times \left[\frac{\mathscr{P}}{k^2 - m_s^2} - i\pi\epsilon(k^0)\,\delta(k^2 - m_s^2)\right] \quad (4.1)$$

The first term in (4.1), which contains the principal value $\mathscr{P}/(k^2 - m_s^2)$, corresponds to the part of $u$-wave (in the region $r \to \infty$) and the second one gives the form of $v$-wave:

$$\underset{(r\to\infty)}{u_s(x)} = \int d^4k \exp(ikx) g_s(k) \frac{\mathscr{P}}{k^2 - m_s^2} \quad (4.2a)$$

$$v_s(x) = -i\pi \int d^4k \exp(ikx) g_s(k)\, \epsilon(k^0)\, \delta(k^2 - m_s^2) \quad (4.2b)$$

Their connection is realised by means of the common function $g_s(k)$.

It should be noticed that by definition of $\mathbf{v}_{\text{reg}} = d\mathbf{r}/dt$ as average front velocity, $u_s(x)$ and $v_s(x)$ may describe the particle behaviour only in average. Therefore the complete theory must be inevitably statistical.

To underline this circumstance, show how one can obtain in our scheme the Planck–de Broglie relation

$$P_\mu = k_\mu \quad (4.3)$$

which is the particular case of the 'guidance' formula (de Broglie, 1956).

Consider a regular solution of (2.4), corresponding to a particle of mass $m_s$. It means that in a proper reference frame the field energy coincides with $m_s$, i.e. $E = m_s$, and the asymptotic behaviour is described by $\psi_s = u_s(x)\, w_s(\xi)$. Consider the flux of these particles, filling all the space. For simplicity we can choose the rest frame of the flux and dispose the particles in the nodes of the cube lattice of dimension $a$, which must be large enough to provide the validity of linear approximation (4.1). If now we introduce the vector

$$\mathbf{d} = (na, ma, la); \qquad n, m, l = 0, 1, 2, \dots \tag{4.4}$$

we can calculate the resulting $U$-field as the following sum:

$$\begin{aligned} U_s(x) &\equiv \sum_{\mathbf{d}} u_s(t, \mathbf{r} + \mathbf{d}) \\ &= \int d^4 k \exp(ikx) \sum_{\mathbf{d}} \exp[i(\mathbf{k}\cdot\mathbf{d})]\, g_s(k) \frac{\mathscr{P}}{k^2 - m_s^2} \\ &\equiv \int d^4 k \exp(ikx)\, G_s(k) \frac{\mathscr{P}}{k^2 - m_s^2} \end{aligned} \tag{4.5}$$

where

$$G_s(k) \equiv \sum_{\mathbf{d}} \exp[i(\mathbf{k}\cdot\mathbf{d})]\, g_s(k) = \left(\frac{2\pi}{a}\right)^3 \delta(\mathbf{k})\, g_s(k) \tag{4.6}$$

By using (4.6) one can find the resulting $V$-field as follows:

$$\begin{aligned} V_s(x) &\equiv -i\pi \int d^4 k \exp(ikx)\, G_s(k)\, \epsilon(k^0)\, \delta(k^2 - m_s^2) \\ &= -i\pi \left(\frac{2\pi}{a}\right)^3 \int dk^0 \exp(-ik^0 t)\, g_s(k^0)\, \epsilon(k^0)\, \delta(k_0^2 - m_s^2) \\ &= b \sin(m_s t) \end{aligned} \tag{4.7}$$

where

$$b = -\frac{\pi}{m_s} \left(\frac{2\pi}{a}\right)^3 g_s(m_s)$$

Thus, in the rest frame of the flux the resulting $V$-field is described by harmonic vibration of frequency

$$k^0 = m_s = E \tag{4.8}$$

The desired relation (4.3) can be obtained from (4.8) by Lorentz transformation.

However, it should be remarked that our proof of the Planck–de Broglie relation is not complete, since from the very beginning we have chosen the regular solution of mass $m_s$. It is not clear why these mass values are preferable, since there are many other solutions of (2.4) with arbitrary masses. This is one of the questions to be solved in a complete statistical theory, permitting one to calculate the transition probability $W[\varphi_1 | \varphi_2]$.

From this point of view, the proposed scalar model is very approximate and cannot be used practically. In future it is proposed to treat the more realistic spinor model.

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